Fractional Order Discrete States-Space System Simulink Toolkit User Guide.

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Abstract
This paper presents User Guide for the Fractional Order Discrete States-Space System Toolkit. The instalation procedure, blocks description and examples is presented.

Keywords: discrete fractional systems, fractional Kalman filter, state feedback control

1 Instalation

To install the toolkit unpack the file fsstX_Y.zip, where X_Y is a version of the toolkit, and add the directory to the Matlab path. The zipped file contains following files:

fsim.c - C-MEX S-function implementation of Fractional States-Space System.

fsim.dll - compiled fsim.c function.

fkf.c - C-MEX S-function implementation of Fractional Kalman Filter

fkf.dll - compiled fkf.c function.

fss.mdl - Simulink library of the toolkit.

slblocks.m - data for Simulink Library Browser.

examples\ex1.mdl - Example of systems simulation.

examples\ex2.mdl - Example of state feedback control.

After instalation the FSST should be seen in Simulink Library Browser. The toolkit two blocks :

”Fractional Order States-Space System” described in Section 2

”Fractional Kalman Filter” described in Section 3

The Toolkit s-function is written in C in order to allow using them in Real Time Workshop and generate code for eg. DS1104 card.

2 Fractional order states-space system s-function

The fractional order states-space system is defined as following (to more detail see [5]):

$$\begin{aligned} \Delta^{\Upsilon} x_{k+1} &= A_d x_k + B u_k & (1) \\ x_{k+1} &= \Delta^{\Upsilon} x_{k+1} - \sum_{j=1}^{k+1} (-1)^j \Upsilon_j x_{k-j+1}. & (2) \\ y_k &= C x_k & (3) \end{aligned}$$

where:

$$\Upsilon_k = \text{diag} \begin{bmatrix} \begin{pmatrix} n_1 \\ k \end{pmatrix} \\ \vdots \\ \begin{pmatrix} n_N \\ k \end{pmatrix} \end{bmatrix} \Delta^{\Upsilon} x_{k+1} = \begin{bmatrix} \Delta^{n_1} x_{1,k+1} \\ \vdots \\ \Delta^{n_N} x_{N,k+1} \end{bmatrix}$$

and $n_1, ..., n_N$ are degrees of system equations.

For real-time simulation there is a requirement of limiting the number of samples in sum operation. It was done by implemented the circular buffer with limited number of samples, given by parameter $Nbuf$.

the $\binom{n}{j}$ symbol is evaluated as:

$$\begin{pmatrix} n \\ j \end{pmatrix} = \begin{cases} 1 & \text{for} j = 0 \\ \frac{n(n-1)...(n-j+1)}{j!} & \text{for} j > 0 \end{cases} \quad (4)$$

what for $j > 0$ can be counted recursively as:

$$\binom{n}{j} = \frac{n}{1} \frac{n-1}{2} \cdots \frac{n-j+1}{j} \quad (5)$$

using this form there is no need to count factorial of very high numbers what is limited by type of used variables .

The S-function fsim have following parameters in turn $A, B, C, N, Ts, Nbuf$.

A, B, C, N are system matrices where A is size Nx,Nx, B is Nx,Nu C Ny,Nx N Nx,1

Ts is a sample time

$Nbuf$ is a width of a circular buffer of past states vectors.

where Nx-number of states, Nu- number of inputs, Ny- number of outputs

3 Fractional Kalman Filter s-function

For system defined in Section 2 the Fractional Kalman Filter is defines as follows (for more detail see [5]):

$$
\begin{aligned}
\Delta^{\Upsilon}\tilde{x}_{k+1} &= A_d\hat{x}_k + Bu_k \\
\tilde{x}_{k+1} &= \Delta^{\Upsilon}\tilde{x}_{k+1} + \sum_{j=1}^{k+1}(-1)^j\Upsilon_j\hat{x}_{k-j+1} \\
\tilde{P}_k &= (A_d + \Upsilon_1)\, P_{k-1}\, (A_d + \Upsilon_1)^T + \\
& \quad +Q_{k-1} + \sum_{j=2}^{k}\Upsilon_j P_{k-j}\Upsilon_j^T \\
K_k &= \tilde{P}_k C^T (C\tilde{P}_k C^T + R_k)^{-1} \\
\hat{x}_k &= \tilde{x}_k + K_k(y_k - C\tilde{x}_k) \\
P_k &= (I - K_k C)\tilde{P}_k
\end{aligned}
$$

where: R is a covariance matrix of output noise ν and Q is a covariance matrix of system noise ω. Both of that noises are assumed to be independent and with zero expected value.

The S-function fsim have following parameters in turn A, B, C, N,$P, Q, R, x_0, Ts, Nbuf$.

A, B, C, N are system matrices where A is size Nx,Nx, B is Nx,Nu C Ny,Nx N Nx,1

P, Q, R, x_0 are FKF matrices where P is size Nx,Nx Q Nx,Nx R Ny,Ny x_0 Nx,1

Ts is a sample time

$Nbuf$ is a width of a circular buffer of past states vectors and past covariance matrices.

The number of system output (rows number of C matrix) is limited to 1 (in future versions will not be limited).

4 Examples

4.1 Example 1: Fractional State-space systems simulation

The aim of this example is to show dependency of the output of the fractional system to the circular buffer length ($Nbuf$). The system used in example is defined by following matrices:

$$A_d = \begin{bmatrix} 0 & 1 \\ -0.1 & -0.2 \end{bmatrix}, N = \begin{bmatrix} 0.7 \\ 1.2 \end{bmatrix} \tag{6}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0.1 & 0.3 \end{bmatrix} \tag{7}$$

The system is simulated for different lengths of the circular buffer ($Nbuf = 10, 20, 50, 100, 300$). Results are shown in Fig. 1.

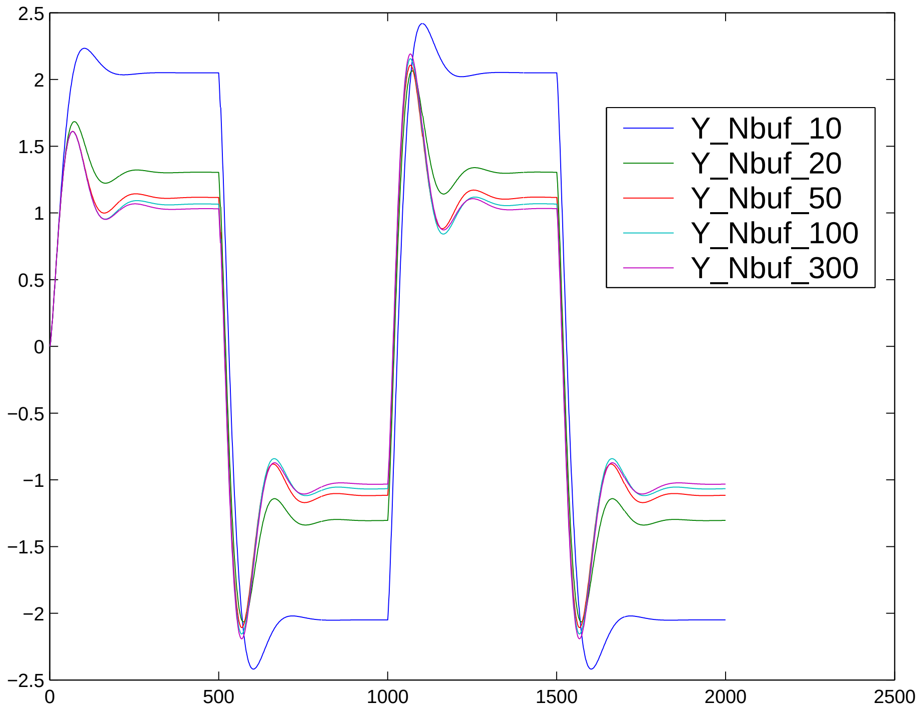

Figure 1: Results Fractional Order States-Space System simulation

4.2 Example 2: Fractional Order States-Space System states feedback control

This example shows the use of states estimation doing by FKF to states feedback control (more detail will be presented at [6]).

The system used in example is defined by following matrices:

$$A_d = \begin{bmatrix} 0 & 1 \\ -0.35 & -0.1 \end{bmatrix}, N = \begin{bmatrix} 0.8 \\ 0.7 \end{bmatrix} \tag{8}$$

$$B = \begin{bmatrix} 0 \\ 1 \end{bmatrix}, C = \begin{bmatrix} 0.3 & 0.4 \end{bmatrix} \tag{9}$$

The control law is defined as:

$$u = u_{ref} - Kx \tag{10}$$

where

$$K = \begin{bmatrix} -0.15 & 0.3 \end{bmatrix} \tag{11}$$

The reference system, for that control law, has a matrix A as follows:

$$A_d = \begin{bmatrix} 0 & 1 \\ -0.2 & -0.4 \end{bmatrix} \tag{12}$$

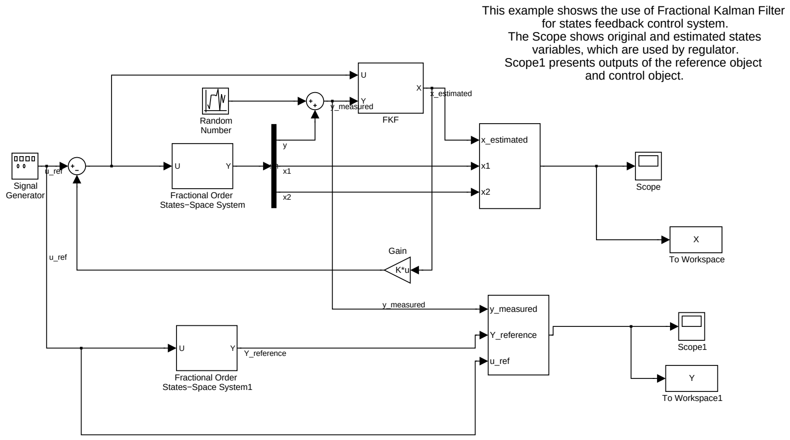

Figure 2: Close-loop feedback control example scheme

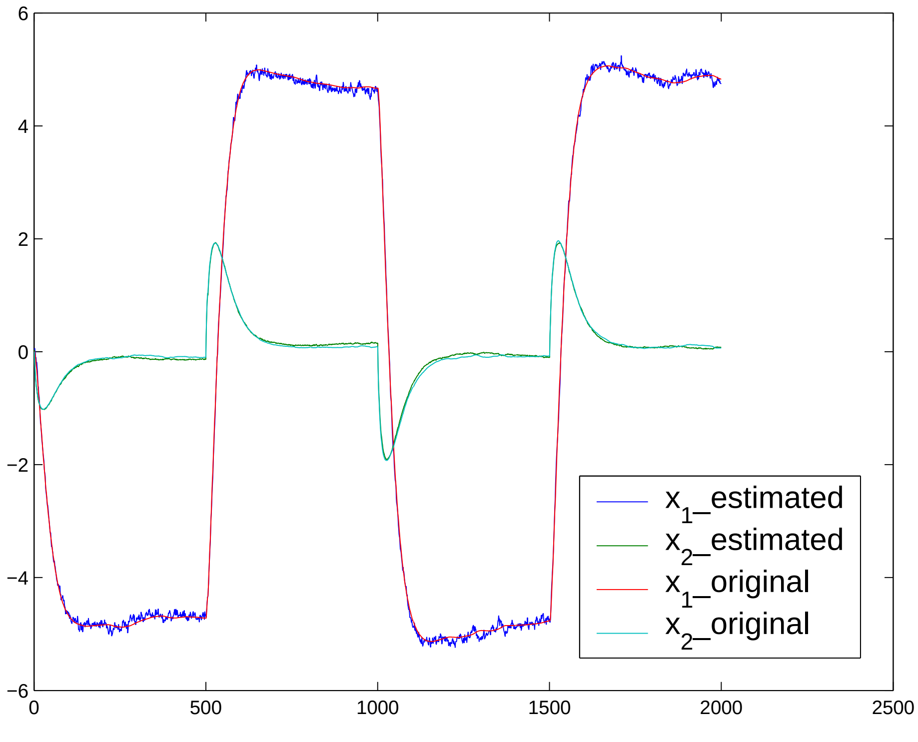

Figure 3: Results of states estimation in close-loop feedback control

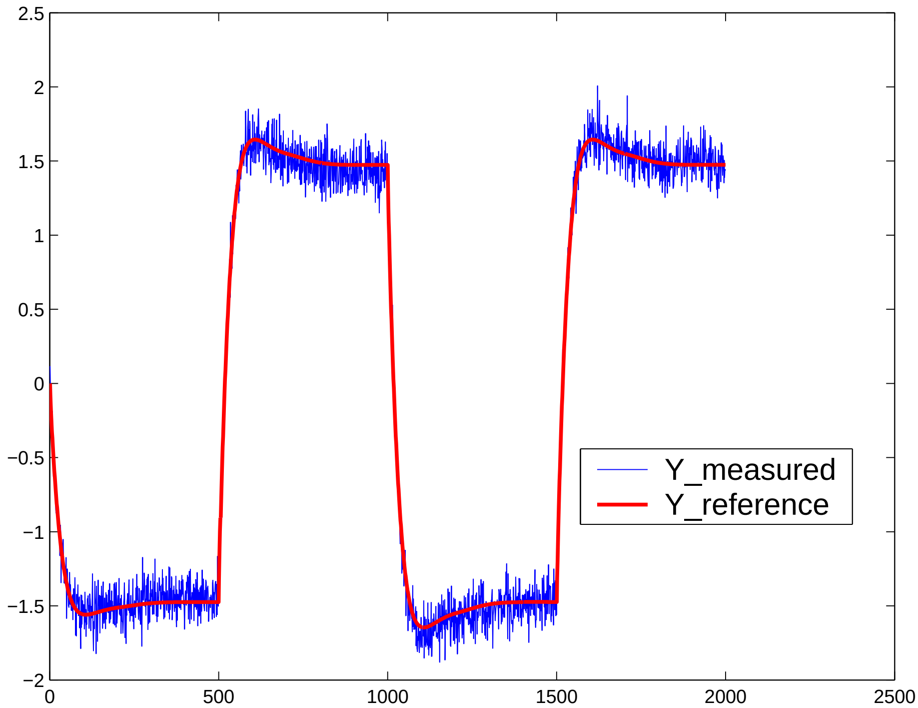

Figure 4: Results of outputs in close-loop feedback control

5 License

The Fractional States-Space Toolkit is free-ware for non-commercial usage.

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